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Quantum Computation with Hot and Cold Ions: An Assessment of Proposed Schemes

D. F. V. JAMES

Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Abstract

We present a brief critical review of the proposals for quantum computation with trapped ions, with particular emphasis on the schemes for quantum computation which relax the stringent requirements for maintaining the ions in the quantum ground state of their collective oscillatory modes.

1. Introduction

Of all the proposed technologies for quantum information processing devices, arguably one of the most promising and certainly one of the most popular is trapped ions. This scheme, discovered by IGNACIO CIRAC and PETER ZOLLER [1], and demonstrated experimentally shortly afterwards by MONROE et al. [2], is currently being pursued by about a half-dozen groups world-wide [3] (for an overview of this work, see, for example refs. [4–6]).

A vital ingredient of trapped ion quantum computing is the ability to cool the ions to their quantum ground state by sideband cooling. Using controlled laser pulses, the quantum state of the ion's collective oscillation modes (i.e. the ions *external* degrees of freedom) can then be altered conditionally on the internal quantum state of the ions' valence electrons, and vice-versa. This allows quantum logic gates to performed. The current state-of-the-art (as of spring, 2000) is that two groups have succeeded in cooling strings of a few ions to the quantum ground state [7–10], and that entanglement of up to four ions has been experimentally reported [11].

The fidelity of the quantum logic gates performed in trapped-ion quantum computers relies crucially on the quantum state of the ions collective oscillatory degrees of freedom. In the original Cirac-Zoller scheme the ions must be in their quantum ground state of these degrees of freedom (the quanta of which are widely referred to as phonons). If the purity of this quantum state were to be degraded by the action of external perturbations (which, given the fact that ions couple to any externally applied electric field, seems quite likely) then the fidelity of quantum operations naturally will suffer. The maintenance of the cold ions in their oscillatory quantum ground state seems at the moment to be the biggest single problem standing in the way of advancing this field. The solution is being tackled in two ways: firstly the understanding and nullification of the experimental causes of the "heating" of the trapped ions, and secondly the investigation of alternative schemes for performing quantum logic operations which relax the strict condition of being in the quantum ground state of the phonon modes. This paper is a brief review and assessment of these schemes.

2. Heating of Ions

The influence of random electromagnetic fields on trapped ions has been analyzed by various authors [12–16]; because this theory impacts on our later discussions, we will give a brief reprise of it here. Consider N identical singly ionized ions, each of mass M, confined in a trap. The trap is assumed to be sufficiently anisotropic, and the ions sufficiently cold that they lie crystallized along an axis of the trap in which the effective trapping potential is weakest, which we shall denote as the x-axis. Because the ions are interacting via the Coulomb force, their motion will be strongly coupled. Their small amplitude fluctuations are best described in terms of normal modes, each of which can be treated as an independent harmonic oscillator [17]. There will be a total of N such modes along the weak axis (we will neglect motion along the directions of strong confinement). We shall number these modes in order of increasing resonance frequency, the lowest ($p = 1$) mode being the center of mass mode, in which the ions oscillate as if rigidly clamped together. In the quantum mechanical description, each mode is characterized by creation and annihilation operators $\hat{a}_p^\dagger$ and $\hat{a}_p$ (where $p = 1, \ldots N$). The ions are interacting with an external electric field $\mathbf{E}(\mathbf{r}, t)$. The Hamiltonian describing this interaction is given by the expression

$$\hat{H} = i\hbar \sum_{p=1}^{N} \left[u_p(t)\, \hat{a}_p^\dagger - u_p^*(t)\, \hat{a}_p \right], \tag{1}$$

where $\hbar$ is Planck's constant divided by 2π and

$$u_p(t) = \frac{ie}{\sqrt{2M\hbar\omega_p}} \sum_{n=1}^{N} E_x(\boldsymbol{r}_n, t)\, b_n^{(p)} \exp\left(i\omega_p t\right). \tag{2}$$

In eq. (2), e is the electron charge, $b_n^{(p)}$ is the n-th element of the p-th normalized eigenvector of the ion coupling matrix [17], ω_p being its resonance frequency, and E_x is the component of the electric field along the weak axis of the trap. In what follows, the center of mass phonon mode ($p = 1$), whose frequency is equal to the frequency ω_x of the Harmonic trapping potential, will have special importance.

The resonant frequencies of the ions' motion is at most a few Megahertz; the wavelengths of the resonant radiation will therefore not be less than 100 meters or so. The separation of the ions is of the order of 10 μm, or 10^7 wavelengths. Thus the spatial Fourier components of the applied field with spatial frequencies of the order of the reciprocal of the ions' separation will be highly evanescent. Therefore, to a very good approximation, one can assume $E_x(\boldsymbol{r}_n, t) \approx E_x(t)$, i.e. the field is constant over the extent of the ion string. Using the fact that $\sum_{n=1}^{N} b_n^{(p)} = \sqrt{N}\,\delta_{p,1}$, the interaction Hamiltonian becomes

$$\hat{H} = i\hbar u_1(t)\, \hat{a}_1^\dagger + \text{h.a.}, \tag{3}$$

where

$$u_1(t) = \frac{ie\sqrt{N}}{\sqrt{2M\hbar\omega_x}} E_x(t) \exp\left(i\omega_x t\right), \tag{4}$$

where $\omega_x \equiv \omega_1$ is the trapping frequency along the x-axis. In other words, spatially uniform fields will only interact with the center-of-mass mode of the ions, which is physical intuitive since some form of differential force must be applied to excite modes in which ions move relative to one another.

The dynamics governed by this Hamiltonian can be solved exactly [16]. The "*heating time*", i.e. the time taken for the occupation number of the center of mass mode to increase by one, is given by the formula (valid in the white noise limit for the spectral density of $E_x(t)$)

$$\tau_N = \frac{M\hbar\omega_x}{Ne^2 E_{\text{RMS}}^2 T}, \tag{5}$$

where E_{RMS} is the root mean square value of $E_x(t)$ and T is its coherence time (we have assumed that $T \gg 2\pi/\omega_x$).

3. Quantum Computing Using "Higher" Phonon Modes

The analysis of the heating of ions presented in the previous section is directly linked to the first, and conceptually most simple method for quantum computing with trapped ions in a manner which avoids the heating problem [7, 16]. Quite simply the "higher" ($p > 1$) modes of the ions' collective oscillations can be utilized in place of the center of mass ($p = 1$) mode originally considered by Cirac and Zoller. The pulse sequence required is exactly that proposed by those authors, with the slight added complication that different laser frequencies (i.e. the sideband corresponding the stretch mode in question) must be employed, and that the laser-ion coupling varies between different ions [17], requiring different pulse durations for different ions. However, as has been pointed out by Saito et al. [19] (in the context of high-temperature NMR experiments) the overall complexity of a computer algorithm involving *classical* control problems of this kind can nullify any speed-up that can be achieved via quantum parallelism.

Experimentally the "higher" modes of the two-ion system are observed to have heating times in excess of 5 μsec, as opposed to heating times of less than 0.1 msec for the center of mass modes [7], confirming that they are indeed well isolated from the influence of external heating fields, and can be used as a reliable quantum information bus.

The heating of the center of mass mode has an important indirect influence. As this mode becomes more and more excited, the wave function of the ions becomes more spatially smeared out, causing a random phase shift of the ions. This effect is analogous to the Debye-Waller effect in X-Ray crystallography [7]. One possible solution for this problem has been proposed [18], namely the use of *sympathetic cooling* by a separate species of ion, allowing the excitation of the center of mass mode to be reduced and kept constant. This scheme however poses the problem of devising a method of loading a trap with an ion of a distinct species and providing a second set of lasers to cool it.

4. Quantum Computation with Macroscopically Resolved Quantum States: The Poyatos, Cirac and Zoller (PCM) Scheme

The essential principle of the scheme proposed by Poyatos, Cirac and Zoller (PCZ) [20] for "hot" ion quantum computation is to create coherent states of the ions' collective oscillations, rather than Fock states. A laser pulse, appropriately tuned, flips the internal state of the ion and simultaneously provides a momentum "kick" to the wave packet of the trapped ion *in a direction which is dependent on the internal state of the ion*. Thus if the ion/qubit is in state $|0\rangle$ it will start to move off in one direction; if it is in state $|1\rangle$ it will start to move in the opposite direction. If it is in a superposition state, then a macroscopic entangled state (or "cat" state) will be created. Because of the strong ion-ion coupling due to the Coulomb interaction, a second ion will also evolve into two spatially dependent wave packets dependent on the state of the first ion (see Fig. 2). If the momentum kick imparted by the initial laser pulse is sufficiently strong then the wave packet associated with the $|0\rangle$ will, after a short time, be spatially distinct from that associated with the $|1\rangle$ state. A laser may then be directed on that distinct wave packet of the second ion, allowing its state to be changed dependent on the state of the first ion (Fig. 3). Once this is done, the motion of the wave packets in the traps restores them to their original positions (Fig. 4) and a third pulse, reversing the effect of the first pulse and nullifying the momentum kick is applied to the first ion, completing the gate operation (Fig. 5).

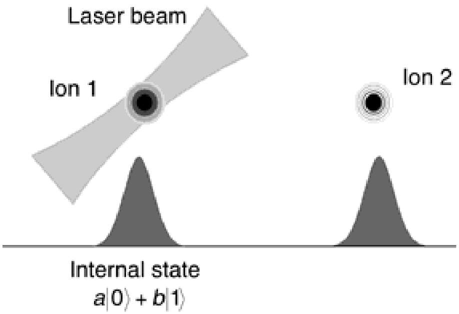

Fig. 1: Schematic picture of the spatial wave packets of the two trapped ion qubits interacting with the laser to give a state-dependent momentum kick.

Fig. 2: As the wave function evolves in time, *both* ions' wave packets become spatially resolved dependent on the state of the first ion. The checkered wave packet is associated with the $|1\rangle$ state, the lined with the $|0\rangle$ state.

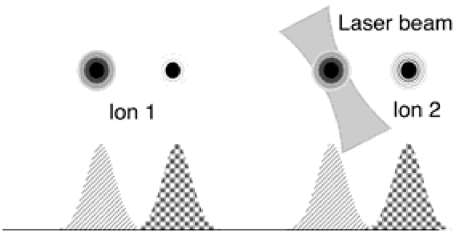

Fig. 3: When the two wave packets are sufficiently separated, a laser is used to flip the state of the second ion *dependent on the state of the first ion*. Note that, in practice, the separation of the wave packets does not have to be greater than the laser's spot size, provided that some controllable difference in the illumination of the two packets is possible.

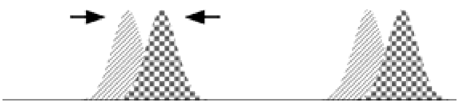

Fig. 4: After the flip, the wave packets oscillate back to their initial spatial positions. The wave packet dynamics in this situation is not simple harmonic motion, since the pulse excites all of the phonon modes.

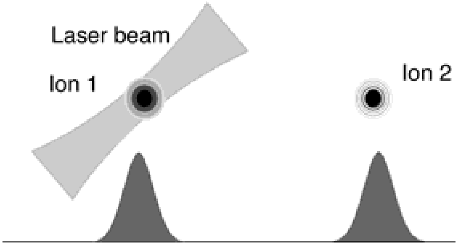

Fig. 5: Finally another laser pulse reverses the effect of the first pulse.

The traveling wave laser pulses which provide the momentum kicks are described by the interaction Hamiltonian

$$\hat{H}_I = \frac{\hbar\Omega}{2}\,\hat{\sigma}^{(+)}\exp\left[i\sum_{p=1}^{N}\eta_p\left(\hat{a}_p + \hat{a}_p^\dagger\right)\right] + \text{h.a.} \tag{6}$$

In this equation Ω is the Rabi frequency, which is proportional to the electric field strength of the laser (see ref. [17] for details) and the operators $\hat{\sigma}^{(+)} \equiv |0\rangle\langle 1|$ and $\hat{\sigma}^{(-)} \equiv |1\rangle\langle 0|$ are respectively the lowering and raising operators for the internal states of the ion (treated as a two level system). In this paper we will be considering the dynamics of one or two ions only, and the context should make it clear as to which of the two ions the operators refer; in some cases subscripts are appended. The constant η_p is the Lamb-Dicke parameter, which characterizes the strength of the coupling between the laser and the p-th oscillatory mode. It varies between different modes and, in general, from ion to ion. It is given by the formula

$$\eta_p = \sqrt{\frac{\hbar k^2\cos^2\theta}{2M\omega_p}}\,b_n^{(p)} \tag{7}$$

where $k = 2\pi/\lambda$ is the laser wavenumber (λ is its wavelength) and θ is the angle between the laser and the x axis (note that, for simplicity, we are assuming single-laser addressing, rather than Raman transitions.)

If this Hamiltonian acts for a time $t_{las} = \pi/\Omega \ll 2\pi/\omega_x$ then the resultant transformation of the state of ion 1 is

$$|\phi'\rangle = \left[\hat{\sigma}^{(+)}\prod_p^N \hat{D}_p(i\eta_p) + \hat{\sigma}^{(-)}\prod_p^N \hat{D}_p(-i\eta_p)\right]|\phi\rangle, \tag{8}$$

where $\hat{D}_p(v)$ is the *displacement operator* for the p-th phonon mode in question [21]. The fact that all of the phonon modes are excited by this operation leads to somewhat complicated dynamics of the excited wave packets. This is alleviated somewhat by the use of slightly different trapping potentials (see [20] for details), which may be realized in small scale traps, in which the electrodes are close to the ions (which could have a detrimental effect on the heating of the ions)[1]). As the number of ions increases this dynamics will becomes more and more complicated, so that one has to wait longer an longer times for the wave packets to re-combine (as in Fig. 4) prior to completion of the

[1]) Another possible method of modifying the ions' collective dynamics is to insert one or more ions of a different mass into the ion chain, as has been investigated in the context of sympathetic cooling by KIELPINSKI et al. [18].

gate. This is phenomenon unfortunately limits this scheme to no more than two or three ions.

For the case of two ions in a trap for which the second phonon mode has twice the resonance frequency of the first mode[2]) the maximum separation of the two wave packets is

$$d_{\max} = \frac{3\sqrt{3}\pi}{2}\,\frac{\hbar\cos\theta}{\lambda M\omega_x}\,. \tag{9}$$

For calcium ions with $\theta = 45^\circ$, λ = 729 nm and $\omega_x = (2\pi)$ 500 kHz, the maximum separation is 3.9 nm, too small for the two wave packets to be resolved optically. However, in practice they do not need to be *completely* separated spatially so that a laser can be focused on one but not the other (as shown in Fig. 3); so long as they are separated somewhat, a laser beam could be applied in such a fashion that one of the wave packets had its internal states flipped while the other received a pulse of the same duration, but different intensity, contrived to leave the internals states effectively unaltered (e.g. a "4π" pulse). Care must however be exercised that laser fields are constant over the spatial extent of each wave packet, otherwise spatial information will become imprinted on the internal degrees of freedom.

Heating effects this scheme by increasing the size of the ions' wave packets. Thus it derives a certain degree of insensitivity to heating by the use of macroscopic effects (i.e. the separation of the ions' wave packets) which are effected but slightly by that heating. However, it is unlikely this scheme is completely immune to the effects of the random electric fields which cause the heating [22]. The random field is equivalent to a random linear variation of the electric potential: As the ions' wave packets move apart, they thus move into regions of different electric potential, which results in a random relative phase shift for the two packets. Furthermore a fair degree of cooling will be necessary in this scheme, if only to reduce the ions' wave-packets to a size that the flipping pulse (Fig. 3) can perform its required operation.

A related scheme for a scalable trapped ion quantum computer was recently proposed by CIRAC and ZOLLER [23]. Two ions in separate but closely spaced harmonic wells can be addressed using laser pulses. When a state-dependent momentum "kick", as described above, is applied to the two ions by a laser pulse, the combination of the wave packets' displacement in the harmonic potential and their mutual Coulomb repulsion results in a state-dependent phase shift, which can be shown to be a universal gate for quantum computation. By using an array of separate traps, each containing one ion, plus a "head" ion which can be moved into the proximity of any of the ions trapped in the array and then used as the control qubit in a quantum gate, one can in principle build a scalable device.

5. Quantum Computation with Virtual Phonons: The Mølmer and Sørensen (MS) Scheme

MØLMER and SØRENSEN (MS) have proposed related techniques for creating both multi-ion entangled states [24] and for quantum computation [25, 26] with ions in thermal motion. The scheme proposed is valid for any mixed state of the ions' collective oscillation modes, and is not confined to thermal equilibrium states. It relies on the virtual excitation of phonon states, in a manner analogous to the virtual excitation of some exited state of an atom or molecule in Raman processes. Laser fields with two spectral

[2]) This can be achieved using a trap whose potential is proportional to $|x|^{5/3}$; see [20].

components detuned equally to the red and to the blue of the atomic resonance frequency are applied to a pair of ions in the trap. The interaction is described by the following Hamiltonian

$$\begin{aligned}\hat{H}_I &= \hbar\Omega \hat{J}^{(+)}\{1 + i\eta(\hat{a}\, e^{-i\omega_x t} + \hat{a}^\dagger\, e^{i\omega_x t})\} \cos(\delta t) + \text{h.a.} \\ &= \hbar\Omega\, e^{i\delta t}\, \hat{J}_x - \hbar\Omega\eta\, e^{i(\delta+\omega_x)t}\, \hat{a}^\dagger \hat{J}_y - \hbar\Omega\eta\, e^{i(\delta-\omega_x)t}\, \hat{J}_y \hat{a} + \text{h.a.}\end{aligned} \tag{10}$$

In this equation $\hat{J}^{(+)} = \hat{J}_x - i\hat{J}_y$ is the collective raising operator for the ions (i.e. the sum of the $\hat{\sigma}^{(+)}$ operators of the different ions) and δ is the detuning of the laser beam from the resonance frequency of the two level system. Note that we have not included subscripts on the mode operators $\hat{a}^\dagger$ and $\hat{a}^\dagger$ or on the Lamb-Dicke parameter η. It is assumed that δ is sufficiently small that only one of the modes contributes to the ions' dynamics; which mode is optional.[3]) When δ is sufficiently large, it is convenient to consider this interaction in terms of an effective Hamiltonian (see appendix), which neglects the effects of very rapidly varying terms. In this case, the effective Hamiltonian is

$$\begin{aligned}\hat{H}_{eff} &= \frac{\hbar\Omega^2\eta^2}{(\delta + \omega_x)}\, [\hat{J}_y\hat{a}, \hat{a}^\dagger \hat{J}_y] + \frac{\hbar\Omega^2\eta^2}{(\delta - \omega_x)}\, [\hat{a}^\dagger \hat{J}_y, \hat{J}_y \hat{a}] \\ &= \frac{\hbar\Omega^2\eta^2}{(\delta - \omega_x)} \left(\frac{2\omega_x}{\delta + \omega_x}\right) \hat{J}_y^2\,.\end{aligned} \tag{11}$$

This interaction is equivalent to a conditional quantum logic gate preformed between the two ions, and can be used to create multiparticle entangled states.

This scheme is very attractive because, while it has the possibility of being scalable to many ions, its operation is *independent* of the occupation number of the phonon modes, and so its fidelity is not degraded by excitation *during* the gate operations themselves. Its chief drawback seems to be the time taken to perform gate operations. In ref. [25] an example is given of population oscillations associated with the above entangling operations in the presence of noise. The Rabi frequency of these oscillations was approximately $4500\omega_x$ (c.f. Fig. 4 of ref. [25], with appropriate change of notation). Given that trap frequencies must be of the order of $\omega_x \sim (2\pi)500$ kHz in order for the ions to be individually resolvable by focused lasers[4]), this implies a gate time of the order of 50 milliseconds. As explained in ref. [26], it is possible to decrease this time by reducing the detuning δ of the laser, at the cost of increasing the susceptibility of this scheme to heating during gate operations, thus necessitating the use of the "higher" (non-center of mass) modes which, as described above, are less susceptible to heating. This scheme is not immune to the Debye-Waller dephasing effect discussed above, and so cooling of the center-of-mass mode into the Lamb-Dicke regime is still required to minimize this cause of decoherence. Nevertheless the MS scheme is a very compelling idea, and has recently been used experimentally to create entangled states of four ions [11].

[3]) In the NIST experiments [11], one of the stretch modes for which $|b_n^{(p)}|$ is independent of n was used. The availability or otherwise of such modes is what limited the number of ions that could be entangled to 4.

[4]) It is not necessary to resolve ions individually for this scheme to be used to create entanglement; however some form of differential laser addressing will be necessary in order to perform quantum computations involving more than two qubits.

6. Quantum Computation via Adiabatic Passages: The Schneider, James and Milburn (SJM) Scheme

The scheme proposed by SCHNEIDER, JAMES[5]) and MILBURN [27] relies on two operations: first the phonon-number dependent a.c. Stark shift introduced by D'HELON and MILBURN [28], and second the use of stimulated Raman adiabatic passage to carry out certain kinds of transitions independently of the occupation number of the phonon mode used as a quantum information bus.

First let us consider the origin of the D'Helon-Milburn shift. The Hamiltonian for a single two-level ion at the node of a detuned classical standing wave is given by the following formula:

$$\begin{aligned}\hat{H}_I &= \frac{\hbar\Omega\eta}{2}\,\hat{\sigma}^{(+)}\left(\hat{a}\,e^{-i\omega_x t} + \hat{a}^\dagger\,e^{i\omega_x t}\right)e^{i\Delta t} + \text{h.a.}\\ &= \frac{\hbar\Omega\eta}{2}\,(\hat{\sigma}^{(+)}\hat{a}\,e^{i(\Delta-\omega_x)t} + \hat{\sigma}^{(+)}\hat{a}^\dagger\,e^{i(\Delta+\omega_x)t}) + \text{h.a.}\,,\end{aligned} \tag{12}$$

where Δ is the laser detuning and, again, we have not been specific regarding which phonon mode is involved. In the limit of large detuning ($\Delta \gg \omega_x$) the *effective* Hamiltonian is (using the result derived in the appendix):

$$\begin{aligned}\hat{H}_{\text{eff}} &= \frac{\hbar\Omega^2\eta^2}{2(\Delta-\omega_x)}\,[\hat{\sigma}^{(-)}\hat{a}^\dagger, \hat{\sigma}^{(+)}\hat{a}] + \frac{\hbar\Omega^2\eta^2}{2(\Delta+\omega_x)}\,[\hat{\sigma}^{(-)}\hat{a}, \hat{\sigma}^{(+)}\hat{a}^\dagger]\\ &\approx -\frac{\hbar\Omega^2\eta^2}{2\Delta}\,(2\hat{n}+1)\,\hat{\sigma}_z = -\frac{\hbar\Omega^2\eta^2}{\Delta}\,\hat{n}(\hat{\sigma}_z + 1/2) + \frac{\hbar\Omega^2\eta^2}{2\Delta}(\hat{n} - \hat{\sigma}_z)\,.\end{aligned} \tag{13}$$

The second term on the right hand side of the final equation represents a level shift, which can be compensated for by detuning the laser. If we choose the duration τ of this interaction to be $\tau = \pi\Delta/\Omega^2\eta^2$, the time evolution is represented by the operator

$$\hat{\mathcal{S}}_t = \exp\left[-i\hat{a}^\dagger\hat{a}(\hat{\sigma}_z + 1/2)\,\pi\right]. \tag{14}$$

This time evolution flips the phase of the ion if the CM mode is in an odd number state *and* the ion is in its excited state. This operation will be performed only on one of the ions (the target qubit) involved in the quantum gate (which we denote by the subscript t). Operations acting on the second ion involved in the gate (the control qubit) will be denoted by the subscript c.

The adiabatic passage [29] required for the gate operation can be realized using two lasers, traditionally called the pump and the Stokes (see Fig. 6). The pump laser is polarized to couple the control qubit state $|1\rangle_c$ to some second auxiliary state $|3\rangle_c$ and is detuned by an amount Δ. The Stokes laser couples to the red side band transition $|2\rangle_c\,|n+1\rangle \leftrightarrow |3\rangle_c\,|n\rangle$, with the same detuning Δ. If the population we want to transfer adiabatically is initially in the state $|1\rangle_c\,|n\rangle$, we turn on the Stokes field (i.e. the sideband laser) and then slowly turn on the pump field (i.e. the carrier laser) until both lasers are turned on fully. Then we slowly turn off the Stokes laser: this is the famous "counter-intuitive" pulse sequence used in adiabatic passage techniques. The adiabatic passage must be performed very slowly. The condition in our scheme is that $T \gg 1/\Omega_{p,n}, 1/\Omega_{S,n}$, where

[5]) In order that the following remarks be read in their correct context, the reader should be aware that the author of the present article was one of the authors of the SJM scheme.

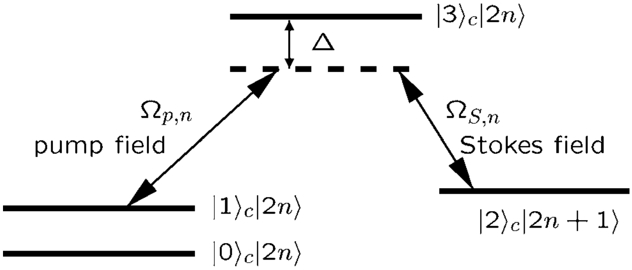

Fig. 6: Illustration of the level scheme of the control ion used to realize the adiabatic passage operations $\mathcal{A}_c^+$ and $\mathcal{A}_c^-$.

T is the duration of the adiabatic passage and $\Omega_{p,n}$ ($\Omega_{S,n}$) are the effective Rabi frequencies for the pump and the Stokes transition, respectively [33]. Using the adiabatic passage we can transfer the population from $|1\rangle_c |n\rangle$ to $|2\rangle_c |n+1\rangle$. To invert the adiabatic passage, we just have to interchange the roles of the pump and the Stokes field. We will denote the adiabatic passage by operators $\mathcal{A}_1^+$ and $\mathcal{A}_1^-$ defined as follows:

$$\begin{aligned} \mathcal{A}_j^+ &: |1\rangle_j |n\rangle \to |2\rangle_j |n+1\rangle \\ \mathcal{A}_j^- &: |2\rangle_j |n+1\rangle \to |1\rangle_j |n\rangle . \end{aligned} \tag{15}$$

The utility of this adiabatic passage scheme is that, despite the fact that the laser transition rates $\Omega_{p,n}$ and $\Omega_{S,n}$ are dependent on the phonon occupation number n, the adiabatic passage using the counter-intuitive pulse sequence is *independent* of n.

These two operations are combined in the sequence shown in Fig. 7 in order to perform quantum gate operations. A detailed breakdown of the operation, including the intermediate states at every stage, is given in [27].

This principle drawbacks of this scheme are two-fold. Because of the adiabatic passage involved, it will of necessity be slow, gates requiring times of the order of a milliseconds. Secondly this scheme (unlike the PCZ and MS schemes) is vulnerable to heating *during* the gate operation.

A further complication is the presence of multiple phonon modes. To take into account their influence, eq. (13) needs to be rewritten as follows:

$$\hat{H}_{eff} = -\frac{\hbar\Omega^2\eta^2}{2\Delta} \sum_{p=1}^{N} \eta_p^2 \left(2\hat{n}_p + 1\right) \hat{\sigma}_z . \tag{16}$$

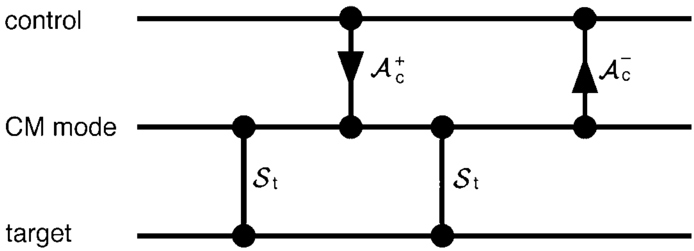

Fig. 7: Schematic illustration of the steps involved in the controlled-Z gate with hot ions. The individual steps are discussed in detail in the text.

The $\hat{\mathcal{S}}_t$ gate will only function as designed when all of the modes except the one to be used as the quantum information bus have zero population. Thus this scheme at best is a means of avoiding the necessity of reducing the population of *every* mode to its quantum ground state; one mode can be left in a mixed state. In this context it is should be noted that the cooling of the "higher" (non center of mass) modes is generally easier than the cooling of the center-of-mass mode, because of their slower heating rates and higher frequencies.

7. Assessment

The various schemes for quantum computation with trapped ions in principle meet many of the criteria for scalable quantum computation technology. Here we discuss the various criteria one by one.

7.1. Initialization

The quantum information register (the ions) and the quantum information bus (the phonon modes) can be initialized reliably using laser cooling and optical pumping. Important aspects of these techniques have already been demonstrated experimentally. The "hot" ion schemes discussed here, if they can be realized experimentally, ease the stringent requirements on preparation of the initial state of the ions collective oscillation modes.

7.2. Gate operations

Quantum logic can be performed using the various schemes outlined above. The common ingredient is laser control of the quantum states of the ions internal and external degrees of freedom, requiring pulses of known duration and strength focused accurately on individual ions. Methods for alleviating the laser focusing problem by altering the ions resonance frequency by various means such as non-uniform electric or magnetic fields have been proposed [6, 34]. The ability to address individual ions with laser beams and control their quantum states has been demonstrated experimentally by two groups using various means [8, 9].

7.3. Isolation from the environment

The internal degrees of freedom of the ions, in which the quantum information is stored, have very long decoherence times (especially when Raman transitions form the basis of the single-qubit operations.) The principal form of environmental disruption suffered by ion traps is disturbance of the motional degrees of freedom, the proposed methods of avoiding this problem being the subject of this article.

The "higher modes" scheme is well isolated from the environment, except for the indirect influence of the Debye-Waller effect. Both the PCM and MS schemes are not intrinsically isolated from the environment, but avoid its influence in various ingenious ways. The SJM scheme will suffer from environmental influences during gate operations unless they can be nullified, for example by using "higher modes".

In all of these schemes the Debye-Waller dephasing effect is a serious fly in the ointment. It can be reduced in two ways: either by reducing the value of the Lamb-Dicke parameter, or by cooling of the ions. The first of these is not a practical alternative, since it will either require an increase in the times required for the gate operations or, in the case of the PCZ scheme, a decrease the wave packet separation. Thus cooling and maintenance of the center of mass mode in a state of low entropy will be required for accurate ion trap quantum computers unless a scheme resilient to Debye-Waller dephasing can be devised.

7.4. Error correction

There is nothing intrinsic that will rule out implementation of fault tolerant quantum computation in ion traps when sufficient numbers of ions become available. Ancilla ions can be prepared in their quantum ground state independent of other ions in the register. The use of multiple stretch modes (there are N-1 such modes in the weak trapping direction), allows quantum gates to be performed in parallel. Read out can be performed at intermediate stages during calculations without destroying the qubit being read, or disturbing other ions in the register unduly (there will be recoil during the read out that has the possibility of excitation of the oscillatory modes).

7.5. Read out

The read-out of the quantum state of ions using a cycling transition has been demonstrated experimentally with high efficiency and reliability [2]. Indeed these experiments are the *only* ones in which high efficiency strong measurement of a single quantum system (as opposed to an ensemble of systems) has been performed.

7.6. Scalability

The ultimate number of ions that can be stored in a string in an ion trap and used for quantum computation is limited by a number of factors. Probably the most important is growing complexity of the sideband spectrum as the number of ions grows. Even in the case of highly anisotropic traps (in which transverse oscillations can be neglected) the number of oscillation modes is equal to the number of ions, and each mode has a distinct frequency, with an infinite ladder of excitation resonances. In addition one has to take into account multi-phonon resonances; the whole leading very complicated structure in frequency space. The extent to which this "spectrum of death" [30] can be understood and exploited, by systematic identification of resonances, careful bookkeeping and tailoring of Lamb-Dicke coefficients remains to be seen. (Indeed, such complex sideband spectra can be advantageous during sideband cooling [35]). Another effect which places an upper bound on the number of ions in a single register is that fact that the spatial separation of the ions decreases $\propto N^{-0.56}$ [31], making their spatial resolution by a focused laser beam more and more difficult. Another, more definite upper bound on the number of ions that can be stored in a linear configuration is the onset of a phase transition to a more complex configuration such as a zig-zag [32]; for traps optimized for quantum computation with singly ionized calcium this occurs at about 170 ions. It is however arguable whether or not the onset of instabilities makes quantum computation impossible.

If only small numbers of ions can be reliably used for quantum computation in a single ion trap, multiple traps will be needed for large scale devices. DeVoe [36] has proposed fabricating multiple elliptical traps, each suitable for a few dozen ions, on a substrate with a density of 100 traps/cm^2. Some form of reliable, high efficiency quantum communication channel to link the multiple traps would need to be implemented [37–39] (see also [23]). An alternative scheme has been proposed by Wineland et al. [6, 40] in which two traps are used. In one a large number of ions is stored in a readily accessible manner (e.g. in an easily rotated ring configuration); each of these ions form the qubits of the register of the quantum computer. When a gate operation is to be performed, the two involved ions are extracted from the storage trap by applying static electric fields in an appropriate controlled manner, and transferred to a separate logic trap where they can be cooled and quantum logic operations can be performed on them. The cooling could be done sympathetically by a third ion of a separate species stored in the logic trap (thereby preserving the quantum information stored in the two logic ions which otherwise would be lost during cooling); in these circumstances either the original Cirac-Zoller scheme or any of the "hot gates" schemes described here can be used as the mechanism for performing the logic; in particular the PCZ scheme, whose principal drawback seems to be its lack of scalability beyond two or three ions, would no longer be at a disadvantage, and given that it is considerably faster than both the MS and SJM schemes, might be attractive.

In conclusion, the variety and richness of the quantum computing schemes that have been devised for ion traps illustrates the great flexibility of this technology. Uniquely amongst the proposals for quantum computing technology, the question for ion traps is not "does it work?" but rather "how far can it be developed?"

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Appendix: Effective Hamiltonians for Detuned Interactions

We start with the Schrödinger equation in the interaction picture, i.e.

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H}_I(t) |\psi(t)\rangle . \tag{17}$$

The *formal* solution of this first order partial differential equation is

$$|\psi(t)\rangle = |\psi(0)\rangle + \frac{1}{i\hbar} \int_0^t \hat{H}_I(t') |\psi(t')\rangle dt' . \tag{18}$$

Substituting this result back into eq. (17), we obtain

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H}_I(t) |\psi(0)\rangle + \frac{1}{i\hbar} \int_0^t \hat{H}_I(t) \hat{H}_I(t') |\psi(t')\rangle dt' . \tag{19}$$

If we assume that the interaction Hamiltonian is strongly detuned, in the sense that $\hat{H}_I(t)$ consists of a number of highly oscillating terms, then to a good approximation the first term on the right hand side of eq. (19) can be neglected, and we can adopt a Markovian approximation for the second term, so that the evolution of $|\psi(t)\rangle$ is approximately governed by the following equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle \approx \hat{H}_{\text{eff}}(t) |\psi(t)\rangle , \tag{20}$$

where

$$\hat{H}_{\text{eff}}(t) = \frac{1}{i\hbar} \hat{H}_I(t) \int \hat{H}_I(t') \, dt' , \tag{21}$$

where the indefinite integral is evaluated at time t without a constant of integration. These arguments can be placed on more rigorous footing by considering the evolution of a time-averaged wave function.

We will now assume that the interaction Hamiltonian consists explicitly of a combination of harmonic time varying components, i.e.

$$\hat{H}_I(t) = \sum_m \hat{h}_m \exp(i\omega_m t) + \text{h.a.}, \tag{22}$$

where h.a. stands for the the hermitian adjoint of the preceding term, and the frequencies ω_m are all distinct (i.e. $m \neq n \Leftrightarrow \omega_m \neq \omega_n$). In this case the effective Hamiltonian $\hat{H}_{eff}(t)$ reduces to a simple form useful in the analysis of laser-ion interactions:

$$\begin{aligned}
\hat{H}_{\text{eff}}(t) &= \sum_{m,n} \frac{1}{i\hbar} \left(\hat{h}_m e^{i\omega_m t} + \hat{h}_m^\dagger e^{-i\omega_m t}\right) \left(\hat{h}_n \frac{e^{i\omega_n t}}{i\omega_n} + \hat{h}_n^\dagger \frac{e^{-i\omega_n t}}{-i\omega_n}\right) \\
&= \sum_{m,n} \frac{1}{-\hbar\omega_n} \left(\hat{h}_m \hat{h}_n e^{i(\omega_m+\omega_n)t} + \hat{h}_m \hat{h}_n^\dagger e^{i(\omega_m-\omega_n)t} - \hat{h}_m^\dagger \hat{h}_n e^{-i(\omega_m-\omega_n)t} - \hat{h}_m^\dagger \hat{h}_n^\dagger e^{-i(\omega_m+\omega_n)t}\right) \\
&= \sum_m \frac{1}{\hbar\omega_m} [\hat{h}_m^\dagger, \hat{h}_m] + \text{oscillating terms} .
\end{aligned} \tag{23}$$

If we confine our interest to dynamics which are time-averaged over a period much longer than the period of any of the oscillations present in the effect Hamiltonian (i.e. averaged over a time $T \gg 2\pi/\min\{|\omega_m - \omega_n|\}$) then the oscillating terms may be neglected, and we are left with the following simple formula for the effective Hamiltonian [6]):

$$\hat{H}_{\text{eff}}(t) = \sum_m \frac{1}{\hbar\omega_m} [\hat{h}_m^\dagger, \hat{h}_m] . \tag{24}$$

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